

# Neutral Hydrogen Tully Fisher Relation: The case for Newtonian Gravity

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## ABSTRACT

Intrinsic luminosities are related to rotation velocities of disk galaxies by Tully Fisher (TF) relations. The Baryonic TF (BTF) relation has recently been explained with Dark Matter and Newtonian Gravity as well as Modified Newtonian Dynamics (MOND). However, recent work has pointed out that the currently used BTF relation ignores the contribution from hot gas and oversimplifies complex galaxy-scale physics. In this Letter, we advocate the use of the Neutral Hydrogen TF (HITF) relationship, which is free from dust obscuration and stellar evolution effects, as a clean probe of gravity and dynamics in the weak field regime. We incorporate the physics of hot gas from supernova feedback which drives the porosity of the Interstellar Medium (ISM). A simple model that includes supernovae feedback, is generalized to include a parametrized effective gravitational force law. We test our model against a catalogue of galaxies, spanning the full range of disks from dwarf galaxies to giant spirals, to demonstrate that a Kennicutt-Schmidt (KS) law for star formation and simple Newtonian gravity is adequate for explaining the observed H I scaling relations. The data rules out MOND-like theories, within the scope of this model.

**Key words:** dark matter — galaxies: kinematics and dynamics — radio lines: galaxies — ISM: atoms — galaxies: fundamental parameters

## 1 INTRODUCTION

If all matter is accounted for, Newtonian gravity and dynamics should provide an adequate description of the motion of celestial bodies in the weak gravity regime. Yet, Zwicky (1937) found that the luminous matter in galaxies (then called Nebulae) could not account for the dynamics in galaxy clusters. This *Dark Matter* (DM) problem was extended to galaxies with the advent of resolved rotation curves (Rubin & Ford 1970). Milgrom (1983) proposed a Modified Newtonian Dynamics (MOND), as an alternative to the DM paradigm. Galactic dynamics provide us a way to test the nature of gravity in the weak field limit and settle one of the most important outstanding questions in physics.

Tully & Fisher (1977) (TF) found a correlation between the global neutral hydrogen (H I) profile widths and absolute optical magnitudes of disk galaxies. This relationship has since then, been well calibrated against Cepheid distances (Sakai et al. 2000) and widely used as a distance indicator (Sakai et al. 2000). Because the absolute magnitudes are essentially a mass proxy and the profile widths are a velocity proxy, this relation implies a connection between the mass

and dynamics. McGaugh et al. (2000) have shown that even though gas dominated galaxies with a low stellar fraction deviate from the TF relation, they fall on a “more fundamental” relation between the total baryonic mass and the rotation velocity. McGaugh (2011) has shown that the BTF relation in gas rich galaxies is consistent with the prediction from MOND. McGaugh (2011) has included only gas rich galaxies in the analysis, to minimize the uncertainty in the baryonic mass introduced by the error in stellar masses. However the selection by gas fraction restricts the range of galaxy masses and rotation velocities. This may also bias the slope as the gas fraction is a function of the total mass. Larson (2010) has also explained the BTF, but within the framework of Newtonian gravity. Hence, the BTF can plausibly be explained in both Newtonian and MOND gravities, using a suitably selected model. Any such test is therefore intrinsically model dependent.

An attempt to explain a TF relation must also take into account, instabilities in the disk (Dalcanton et al. 1997), resulting star formation and supernova feedback (Silk 1997). Gnedin (2011) has pointed out that BTF only accounts for stars, molecular, and atomic gas, while the contribution of ionized gas is almost universally missed. Such a situation has led Foreman & Scott (2011) to comment that the claimed

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Cold DM prediction for the BTf, by McGaugh (2011), is a gross oversimplification of the complex galaxy-scale physics involved. Our work, for the first time, accounts for the role of supernova driven hot gas in determining the HI scaling relations. In fact Silk (1997) has explained the TF relation in the B-band, which traces recent star formation, by considering self regulated star formation. However, the optical photons are strongly affected by dust obscuration and inclination, which may significantly bias the TF relations. Using Near Infrared TF relations (Giovanelli et al. 1997; Masters et al. 2006) can minimize dust effects and provide tighter correlations. The alternative is to use a relation between HI masses and HI profile widths, which we henceforth refer to as the HI Tully Fisher (HITF) relation. This relation is far less sensitive to obscuration as the universe is largely transparent at radio wavelengths (Khedekar & Chakraborti 2011). The HI signal originates from the ISM and is therefore independent of the stellar ages in the galaxy.

Any model which explains this relation must also explain (Chakraborti 2011) the observed (Rosenberg & Schneider 2003; Begum et al. 2008b) scaling relation between HI masses and areas of disk galaxies. In this Letter we further develop an analytic model by Silk (1997) and Chakraborti (2011) to derive the HI masses of disk galaxies, taking into account the competition between gravitational instabilities and mechanical feedback from supernovae. We also generalize the model to include arbitrary  $(\sqrt{M}/r)^\gamma$  laws of gravity and test it against our catalogue of galaxies. The HITF data spans the full range of disks from dwarfs galaxies observed with the Giant Metrewave Radio Telescope (GMRT) (Begum et al. 2008b), to giant spirals observed with the Arecibo (Gurovich et al. 2010). The HI mass vs area data is from Rosenberg & Schneider (2003). We conduct a joint likelihood analysis of these data sets within the scope of our model to demonstrate the validity of Newtonian gravity at galactic scales, and also rule out MOND.

## 2 HI SCALING RELATIONS

HI scaling relations in galaxies have been known for the past two decades. Briggs & Rao (1993) had already found a weak scaling ( $\Delta V \propto M_{\text{HI}}^{-0.3}$ ) of the velocity width ( $\Delta V$ ) with the HI mass ( $M_{\text{HI}}$ ). This however implied a strong dependence of  $M_{\text{HI}}$  on  $\Delta V$  with a power law index between 2.1 and 3.6 (Briggs & Rao 1993). The optical TF relation, which measures stellar mass, breaks down at the low luminosity end (McGaugh et al. 2000). However no such breaks have been reported in the HI scaling. We are now in a position to extend the HITF to the full range of galaxy masses for the first time and show in this work that it is consistent with a single scaling relation.

Scaling relations between  $M_{\text{HI}}$  and the surface area of galaxies also have a long history. Giovanelli & Haynes (1983) reported that HI sizes and masses were found to be correlated in the same manner, irrespective of whether they were HI rich or HI poor. Optical sizes and HI masses were also seen to be correlated (Haynes & Giovanelli 1984; Verheijen & Sancisi 2001). Rosenberg & Schneider (2003) reported that  $M_{\text{HI}}$  and surface areas of ADBS (Rosenberg & Schneider 2000) galaxies are consistent with

a nearly constant average HI surface density of the order  $\sim 10^7 M_\odot \text{ kpc}^{-2}$ . Chakraborti (2011) has recently explained this surprising correlation, as the result of self-regulated star formation, driven by the competition between gravitational instabilities in a rotationally supported disk and mechanical feedback from supernovae.

## 3 GENERALIZED MODEL

Following Silk (1997), we consider a rotationally supported gas disk, fragmenting and forming stars due to gravitational instability. In such a scenario the asymptotic velocity can be related to the enclosed dynamical mass ( $M_d$ ). However this requires the knowledge of an effective force law for gravity in the weak field limit. Since we wish to determine the nature of gravity at these scales, it is necessary not to use a particular variant, but a parametrization which can encompass a wide range of theories including Newtonian gravity and MOND<sup>1</sup>. We therefore, parametrize the gravitational force ( $F$ ) at a radius  $r$  as,

$$F \propto \left( \frac{\sqrt{M_d}}{r} \right)^\gamma, \quad (1)$$

where  $\gamma$  is a free parameter which is to be determined from observations. Here,  $\gamma = 2$  gives back the simple Newtonian case, while  $\gamma = 1$  gives the familiar MOND case in the low acceleration regime relevant at galactic scales.

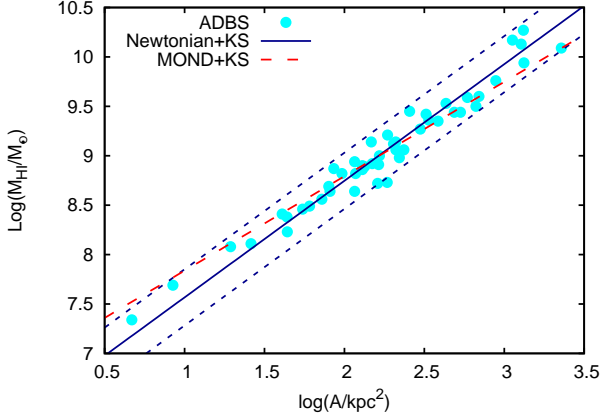
To relate the gravitational force to a rotation velocity, one needs a prescription for  $M_d$ . Garcia-Appadoo et al. (2009) points out that the observed dynamical mass scales as cube of the optical radius. Loeb & Weiner (2011) have shown that dark matter particles interacting through a Yukawa potential could provide a natural explanation for a characteristic density in dark matter dominated halos. We exploit this observed (Garcia-Appadoo et al. 2009) relation  $M_d \propto R^3$ , where  $R$  is the size of the HI disk, to express the rotation velocity as,

$$V \propto R^{(2+\gamma)/4}. \quad (2)$$

Thus the dependence of the asymptotic rotation velocity on disk size is a function of  $\gamma$  and therefore is a probe of the effective gravitational force law in the low acceleration case.

Chakraborti (2011) has argued that the observed correlation between HI mass and surface area may be the result of self regulated star formation. Even when a disk galaxy is driven away from this fundamental line, self regulation of the porosity (driven by supernova feedback (McCray & Snow 1979) of the ISM and gravitational instability (Safronov 1960; Toomre 1964) of the star forming disk, can control the star formation rate. This ensures that the star forming disk is always marginally unstable. Under these considerations Silk (1997) has shown that the star formation rate (SFR) scales as,  $\text{SFR} \propto V^{5/2}$ , where  $V$  is the asymptotic rotation velocity of the disk.

<sup>1</sup> MOND can be thought of as a modification of either the inertial term ( $F = \mu m a$ ) or of the effective gravitational force term ( $F = F_{\text{Newton}}/\mu$ ) by  $\mu(a/a_0)$ , where  $a_0$  is the acceleration scale below which the dynamics is modified. At the low acceleration regime, relevant for disk galaxies, they result in the same rotation curves (Sanders & McGaugh 2002).



**Figure 1.**  $M_{\text{HI}}$  vs  $A$  data for ADBS galaxies (cyan balls) from Rosenberg & Schneider (2003). Statistical uncertainties are comparable to the sizes of the balls. Blue solid (and dotted) line shows the Newtonian+KS model (and  $2\sigma$  scatter region) with  $\gamma = 2$  and  $\beta = 1.4$  put in Eqn. 5. Red dashed line shows the MOND+KS model with  $\gamma = 1$  and  $\beta = 1.4$ .

We can now express the surface density of the SFR ( $\Sigma_{\text{SFR}}$ ) as  $\Sigma_{\text{SFR}} \equiv \text{SFR}/(\pi R^2) \propto V^{5/2} R^{-2}$ . Substituting for the rotation velocity ( $V$ ) as a function of  $R$  from Eqn. 2, we have

$$\Sigma_{\text{SFR}} \propto R^{(5(2+\gamma)/8)-2} \propto R^{(5\gamma-6)/8}. \quad (3)$$

This relates the surface density of SFR to the size of the galaxy. However, the  $\Sigma_{\text{SFR}}$  is also related to the surface density of cold gas according to the KS law (Kennicutt 1998). If the cold gas in a galaxy is dominated by HI, then we can use the KS law to express the SFR surface density as  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{HI}}^\beta$ , where Kennicutt (1998) gives the power law index  $\beta$  a value of  $1.4 \pm 0.15$ .

Eliminating  $\Sigma_{\text{SFR}}$ , we get the HI surface density as  $\Sigma_{\text{HI}}^\beta \propto R^{(5\gamma-6)/8}$ . Integrating the surface density over the size of the galaxy, we get

$$M_{\text{HI}} \equiv \pi R^2 \Sigma_{\text{HI}} \propto R^{((5\gamma-6)/(8\beta))+2}. \quad (4)$$

Expressing this as a function of the surface area, we get

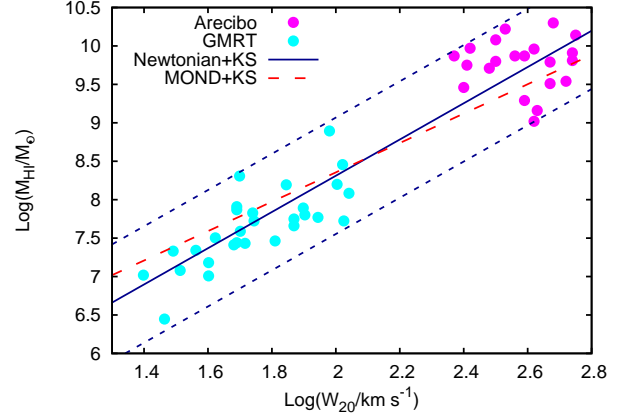
$$M_{\text{HI}} = C_A A^{((5\gamma-6)/(16\beta))+1}, \quad (5)$$

where  $C_A$  is a constant of proportionality. This generalizes the result ( $M_{\text{HI}} \propto A^{1.18}$  for Newtonian gravity and KS law slope of 1.4) from Chakraborti (2011) to arbitrary KS law slopes and generalized effective gravitational force laws. Khedekar & Chakraborti (2011) have shown that such a relation depends only weakly on the metallicity of the galaxy. The power law relates the HI mass of a galaxy to its size and can be directly compared to the data (See Fig. 1).

We can now use the scaling in Eqn. 2 between  $V$  and  $R$  to find  $M_{\text{HI}}$  as a function of the galaxy size. This gives us the HI mass as

$$M_{\text{HI}} = C_V V^{(5\gamma+16\beta-6)/(2\beta(2+\gamma))}, \quad (6)$$

where  $C_V$  is another constant of proportionality. This shows that the observed slope of the HITF relation is a probe of both the KS law and the law of gravity. For Newtonian gravity and KS law slope of 1.4 we have  $M_{\text{HI}} \propto V^{2.36}$ . This can now be compared directly with the data (See Fig. 2), but a



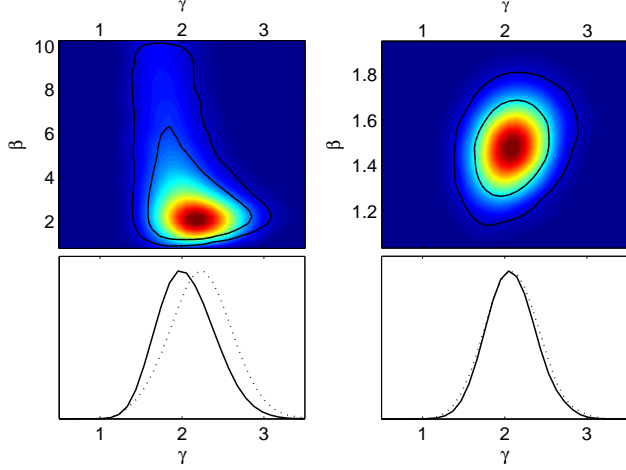
**Figure 2.**  $M_{\text{HI}}$  vs inclination corrected  $W_{20}$  data for galaxies from Arecibo (magenta balls) and GMRT (cyan balls). See text for sources of data. Statistical uncertainties are comparable to the sizes of the balls. Blue solid (and dotted) line shows the Newtonian+KS model (and  $2\sigma$  scatter region) with  $\gamma = 2$  and  $\beta = 1.4$  put in Eqn. 6. Red dashed line shows the MOND+KS model with  $\gamma = 1$  and  $\beta = 1.4$ .

prior knowledge of the KS law slope will be required to determine the effective gravitational force law. This shortcoming can be overcome by simultaneously comparing Eqn. 6 to the HITF data and Eqn. 5 to the HI mass vs area data. The ratio of the two slopes is a function of  $\gamma$ , but independent of  $\beta$ . Hence, determining both slopes from observations can break the  $\gamma - \beta$  degeneracy and determine  $\gamma$  irrespective of our prior knowledge of  $\beta$ . Being able to explain both observed relations for plausible values of these parameters is a crucial reality check for our model.

#### 4 CATALOGUE OF GALAXIES

The Arecibo Dual-Beam Survey (Rosenberg & Schneider 2000) (ADBS) has conducted a “blind” survey of  $\sim 430$  deg<sup>2</sup> of sky and detected the HI signal in 265 galaxies. Interferometric mapping of 84 galaxies was carried out with the NRAO’s Very Large Array (VLA) as most of the ADBS galaxies were unresolved at the resolution of the Arecibo. Accurate sizes of 50 of them were compiled by Rosenberg & Schneider (2003) to tabulate  $M_{\text{HI}}$  and  $A$  (HI cross section with column density above  $2 \times 10^{20}$  cm<sup>-2</sup>) of individual ADBS galaxies. The observed sizes span 3 orders of magnitude ( $0.5 > \log(A/\text{kpc}^2) > 3.5$ ) and provide the data for the HI mass vs area relation. For the following analysis, we have used only objects with large projected areas ( $\log(A/\text{kpc}^2) > 2$ ) as the smaller galaxies may have systematically uncertain sizes limited by the resolution of the present surveys. Fig. 1 presents the observed data and compares it with the fiducial models.

Gurovich et al. (2010) presented a sample of local ( $D < 60$  Mpc) field galaxies with accurate 21 cm observations to construct a BTFR relation. We select the galaxies listed by Sakai et al. (2000) and used by Gurovich et al. (2010). All these galaxies have accurate Cepheid distances (Sakai et al. 2000) determined using the *Hubble Space Telescope*  $H_0$  Key Project and 21 cm Arecibo observations from



**Figure 3.** The hotter (cooler) colours denote regions with higher (lower) mean likelihoods. The solid (dotted) lines denote the  $1\sigma/2\sigma$  confidence limits of the marginalized (mean) likelihoods. The figures on the right include a Gaussian prior on  $\beta$ , while the figures on the left do not include such an external prior. Note that the value of  $\gamma=1$ , corresponding to the case of the MOND gravity, is completely ruled out, while the Newtonian gravity ( $\gamma=2$ ) gives an excellent fit to the data.

Giovanelli et al. (1997). For all these galaxies we use  $M_{\text{HI}}$  and  $W_{20}$  (the width at which the HI profile drops to 20% of its maximum value) as listed by Gurovich et al. (2010), inclination corrected following the method of Giovanelli et al. (1997). Following Tully & Fisher (1977) this is commonly used as a proxy for rotation velocity in radio astronomy<sup>2</sup>. This provides the data for the high mass range in the HITF relation.

To accurately determine the slope of the HITF relation, this catalogue needs to be supplemented with data on low mass galaxies as well. The Faint Irregular Galaxies GMRT Survey (FIGGS) conducted by Begum et al. (2008b) aims at characterizing the neutral ISM properties of faint, gas-rich dwarf galaxies through 21 cm GMRT observations. This provides us with the  $M_{\text{HI}}$  and  $W_{20}$  (again inclination corrected following Ref. (Giovanelli et al. 1997)) measurements at the low mass end of the HITF relation. At the low velocity end, line broadening due to velocity dispersion from turbulent motion in the HI, may play a significant role. We correct for this effect following the prescription from Tully & Fouque (1985)<sup>3</sup>. Fig 2 displays the observational HITF data and compares it with the fiducial models.

<sup>2</sup> It is possible to infer a rotation velocity  $V_{\text{rot}}$  by fitting, say a tilted ring model (with a poorly determined inclination) to the HI data cube, but this is model dependent. Begum et al. (2008a) confirm that  $W_{20}$  correlates better with mass indicators than  $V_{\text{rot}}$ . In the rest of the analysis we replace  $V_{\text{rot}}$  with its observational proxy  $W_{20}$ .

<sup>3</sup> This may introduce a systematic uncertainty. However, according to Eqn. 5 of Silk (2005), a 10% variation in the gas density, leads to a  $\sim 1\%$  change in the gas dispersion velocity. This will introduce a  $\lesssim 2\%$  systematic error in the corrected velocity width, which is small compared to the statistical uncertainty of  $\sim 10\%$ .

## 5 MCMC ESTIMATION

In order to obtain an effective law of gravity at the galactic scales using the data on  $M_{\text{HI}}$ ,  $A$  and  $v$ , we consider the two scaling relations –  $M_{\text{HI}}$  vs  $A$  and  $M_{\text{HI}}$  vs  $v$ , as expressed through Eqn. 5 and 6 respectively. We compute  $\chi^2$  for the data as a sum,  $\chi^2 = \chi_A^2 + \chi_v^2$ , one for each for the relations –  $M_{\text{HI}}$  vs  $A$  and  $M_{\text{HI}}$  vs  $v$ , respectively. For each  $\chi^2$ . The weight,  $\sigma$ , is computed by adding the intrinsic scatter and observed error in quadrature.

We then compute the likelihoods,  $\exp(-\chi^2/2)$  in the four dimensional parameter space of  $\{C_A, C_V, \beta, \gamma\}$ . We perform Markov Chain Monte Carlo (MCMC) runs<sup>4</sup> in the using these likelihoods. Of these parameters, we eventually marginalize over the nuisance parameters  $C_A$  and  $C_V$ . Note that we do not assume a completely deterministic form for the KS law, but require only a simple power law relation, which is eventually marginalized when quoting our final results on  $\gamma$ .

The results of our likelihood analysis is shown in Fig. 3. The top figures displays the  $1\sigma$  and  $2\sigma$  constraints in the  $\beta$ – $\gamma$  plane. The top left figure shows the likelihoods without any external prior on  $\beta$ , while the top right figure shows the results after inclusion of a Gaussian prior of  $\beta = 1.4 \pm 0.15$  as obtained by Kennicutt (1998) through independent observations. The corresponding figures at the bottom show the marginalized (over  $\beta$ ) 1-D distribution on  $\gamma$ .

Our MCMC exploration of the parameter space, without the priors on the star formation law, reveals that even though  $\beta = 2.01^{+4.23}_{-1.26}$  is not well constrained, the effective law of gravity is highly constrained with  $\gamma = 2.06 \pm 0.34$ , consistent with Newtonian gravity ( $\gamma = 2$ ) but clearly ruling out MOND ( $\gamma = 1$ ). At the same time, the best fit region in the  $\beta$  –  $\gamma$  plane also includes the region with KS law slope of  $\beta = 1.4 \pm 0.15$  from Kennicutt (1998). Hence our result confirms that the observed HI scaling relation is not only consistent with Newtonian gravity but also with the KS law prescription for star formation<sup>5</sup>. We find that both with and without an external prior on  $\beta$ , our model can conclusively rule out the MOND model of gravity; the constraints on  $\gamma$  get slightly better after including the prior on  $\beta$ . This shows that our results are insensitive to the prior used on the KS law, as expected from Section 3. The likelihood analysis yields values of  $\gamma = 2.08 \pm 0.29$  after including an external prior on  $\beta$ .

## 6 DISCUSSIONS

TF relations are being increasingly discussed (and also criticised) in the context of weak field tests of gravity. In this letter we have suggested the simultaneous use of two HI

<sup>4</sup> An MCMC algorithm is useful for sampling the likelihoods from a high dimensional probability distribution based on constructing Markov chains that have the desired distribution as their equilibrium distribution, *i.e.*  $\exp(-\chi^2/2)$ ; see Lewis & Bridle (2002) for more details.

<sup>5</sup> Roychowdhury et al. (2011) have found that when comparing  $\Sigma_{\text{HI}}$  to the H $\alpha$  emission, the KS law is steeper ( $\beta \sim 2$ ), which is also consistent with our result. A summary of the impact of different star formation laws on galaxy formation is given by Lagos et al. (2010).

scaling relations for this purpose. Our test is free from some of the systematics and selection bias that affect the existing methods. Our model incorporates, for the first time, the physics of supernova feedback driven hot gas into this analysis. We have shown that a simple model of self regulated star formation accounts for the observed HI scaling relations in disk galaxies.

The crucial ingredients which make the model agree with the data are the star formation law and the effective law of gravity in the weak field limit. Earlier work shows that rotation curves and single scaling relationships, such as the BTF, can be explained both within the DM (Larson 2010) and MOND (McGaugh 2011) paradigms. Hence, a new method was required to use galactic dynamics for a weak field test of gravity. Our work shows that the HITF and the M-vs-A relations taken together can constrain the effective law of gravity relevant on the galactic scale. The data rules out MOND at more than 99.9% confidence at galactic scales within the scope of this model. We show that our model suffices to explain the data using a simple KS law prescription for star formation and Newtonian gravity. This is a triumph for the DM paradigm with Newtonian gravity in the low acceleration regime.

With Extended-VLA follow up of ALFALFA (Giovanelli et al. 2005) detected galaxies and the advent of high sensitivity, high resolution, next generation radio telescopes such as the Square Kilometer Array, the number of HI detected galaxies is set to grow rapidly (Khedekar & Chakraborti 2011). Our study implies that the soon to be observed scaling relations between HI masses of these galaxies, their rotation velocities and sizes will provide an excellent probe for the nature of gravity and dynamics in the weak field limit.

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